Translating English into (in)equalities

"Inside the sphere of radius 3 centered @ (1,2,0)"

$$(x-1)^2+(y-2)^2+z^2 \leq 3^2$$

"Below the graph of $f(x,y)=5-x^2-y^2$ in the first octant"

$$z=5-x^2-y^2$$

$$z \leq 5-x^2-y^2$$

$x \geq 0 \quad y \geq 0 \quad z \geq 0$

15.8.35

$z=\sqrt{x^2+y^2}$ $\qquad x^2+y^2+z^2=1$

i.e. $z=r$.

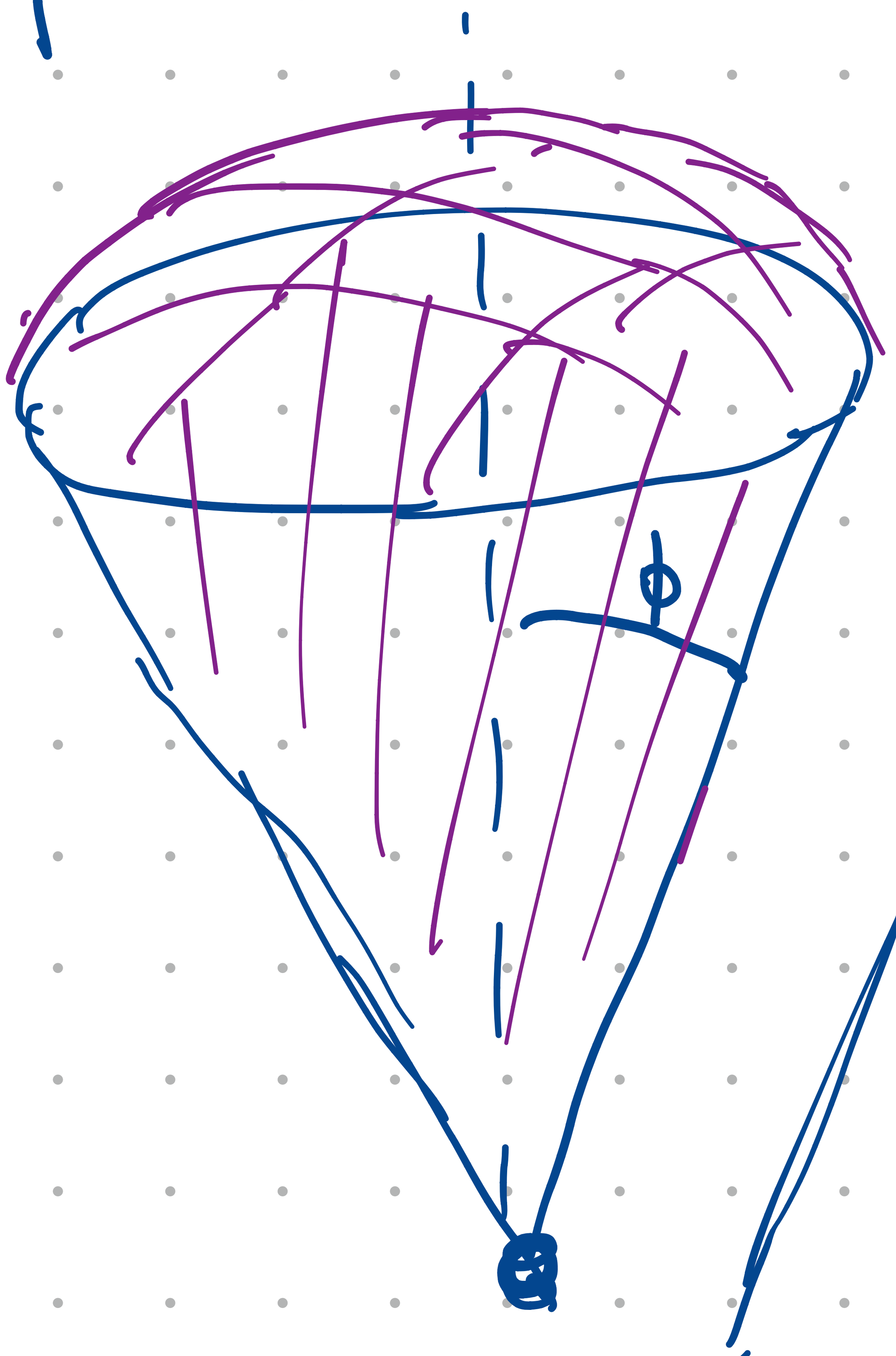
$x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$

$$\begin{cases} z\geq\sqrt{x^2+y^2} & ① \\ x^2+y^2+z^2\leq 1 & ② \end{cases}$$

$\}$ spherical



$$\begin{cases} \boxed{0\leq\theta\leq 2\pi} \\ \boxed{0\leq\phi\leq\pi} \\ \boxed{0\leq\rho} \\ r/z=\boxed{\tan\phi\leq 1} & ① \\ \boxed{\rho^2\leq 1} & ② \end{cases}$$

$0<1$

$0\leq\pi/4$

$$\iiint \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

ρ: $0 \le \rho, \quad \rho^2 \le 1$

So ρ bounds are 0 to 1.

ϕ: $0 \le \phi \le \cancel{\pi}$ and $\underbrace{\arctan 1}_{=\pi/4}$

redundant

So ϕ bounds 0 to $\pi/4$

θ: $0 \le \theta \le 2\pi$

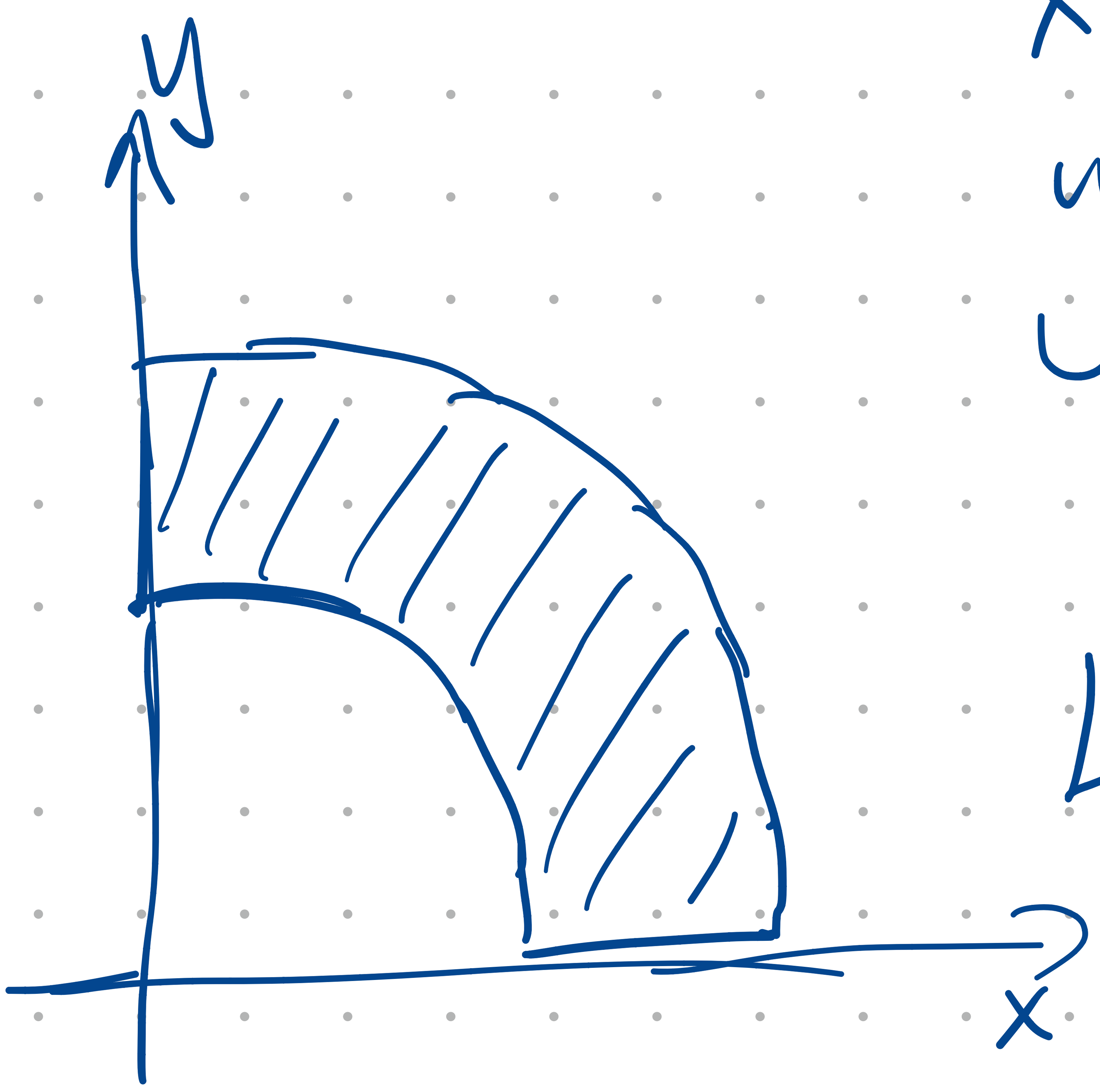
15.9.13 "between the circles ...."

$$1 \leq x^2 + y^2 \leq 2$$

"in the first quadrant"

$x \geq 0$

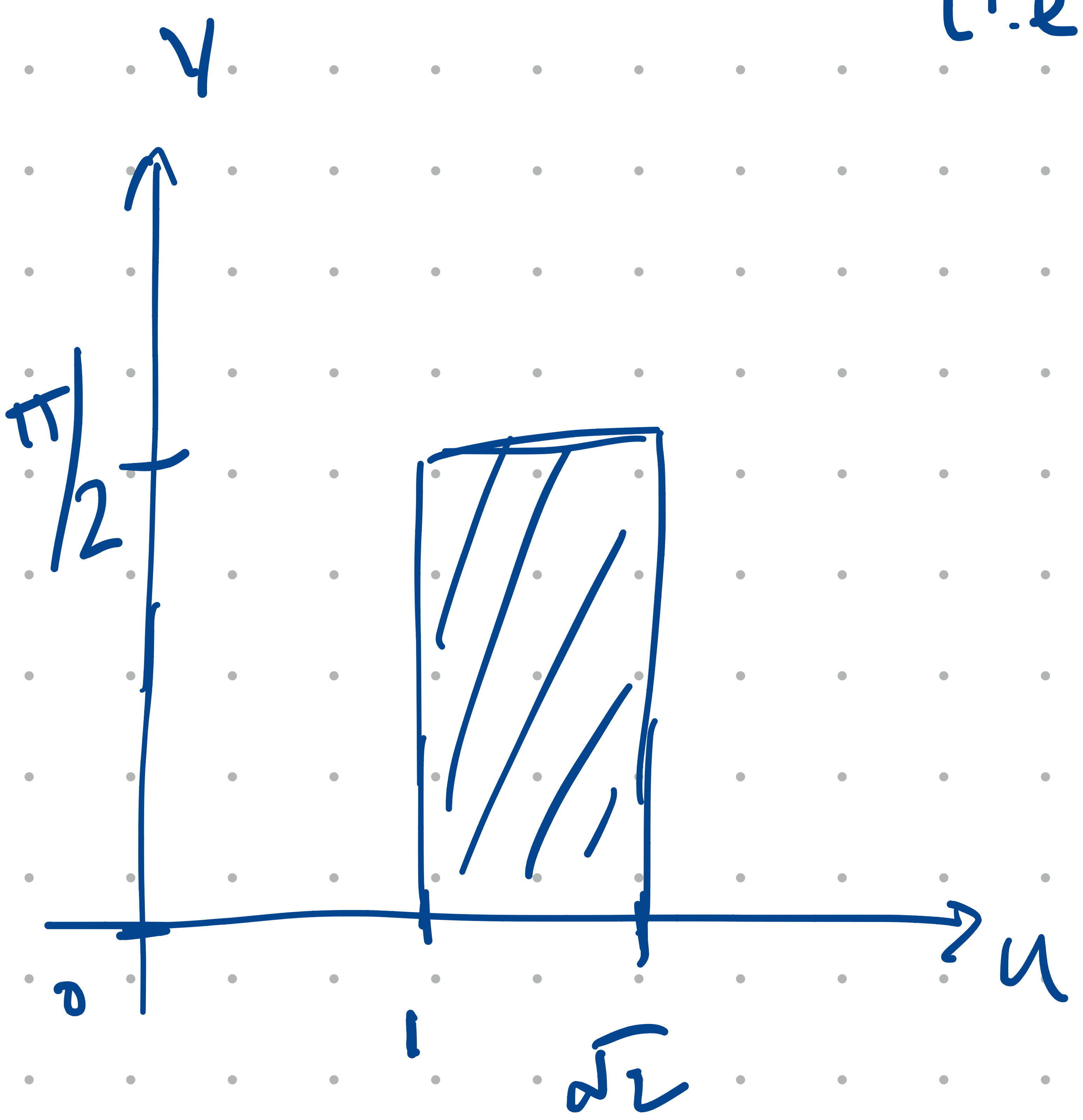
$y \geq 0$



Let $x = u \cos v$

$y = u \sin v$

(i.e. polar coords.)



Example for 15.9

Let R be the trapezoidal region with corners $(1,0)$, $(2,0)$, $(0,2)$, $(0,1)$.

Compute $\displaystyle\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$

Natural candidates for u,v: $u = y-x$, $v = y+x$.

$$x = \frac{v-u}{2}$$
$$y = \frac{v+u}{2}$$

What happens to the integrand?

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \left|\det\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\right|$$
$$= \frac{1}{2}.$$

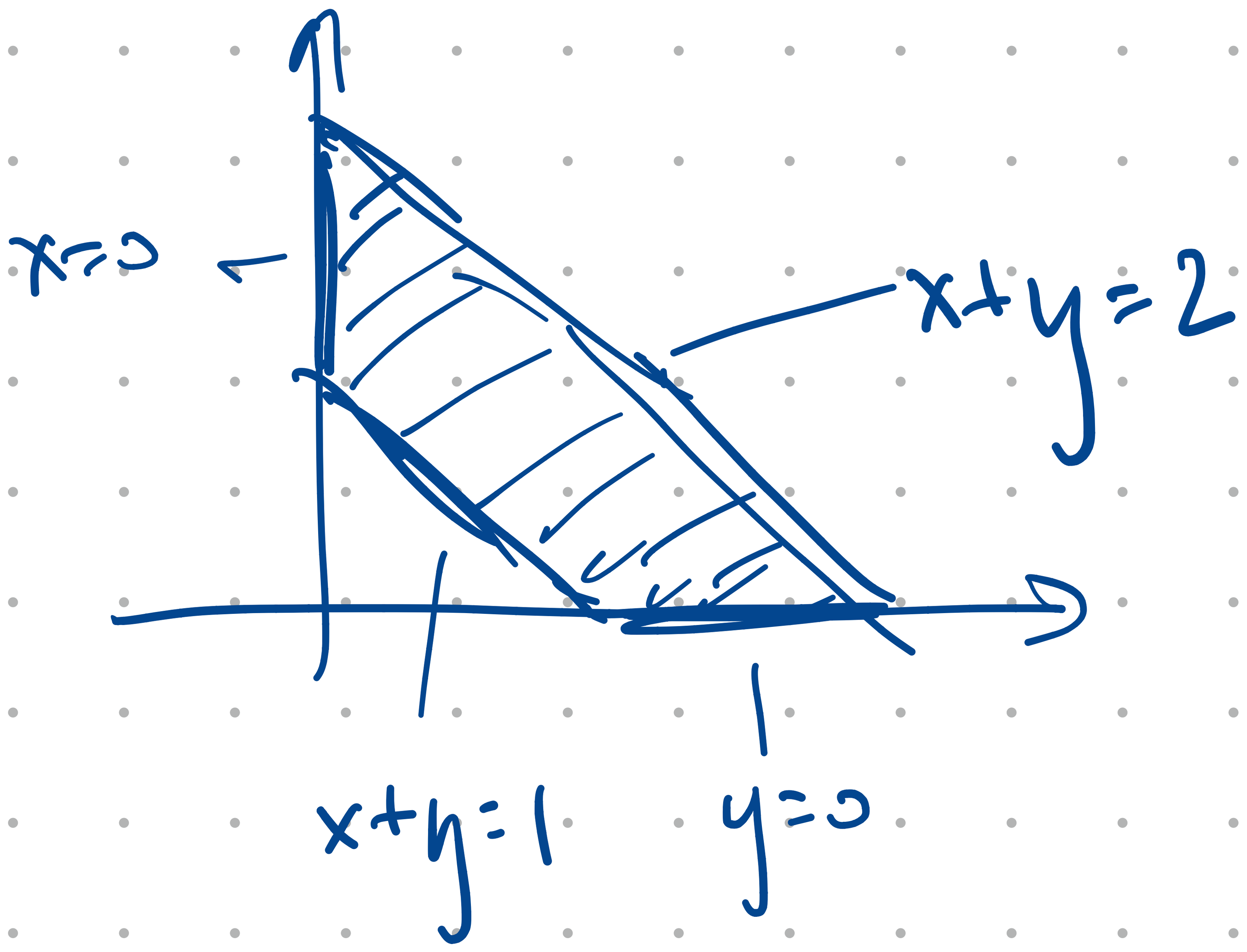
So new integrand is

$$\cos\left(\frac{u}{v}\right)\cdot\left(\frac{1}{2}\right) du\,dv$$

(or $dv\,du$)

What happens to the region of integration:

Describe xy region algebraically w/ ineqs:

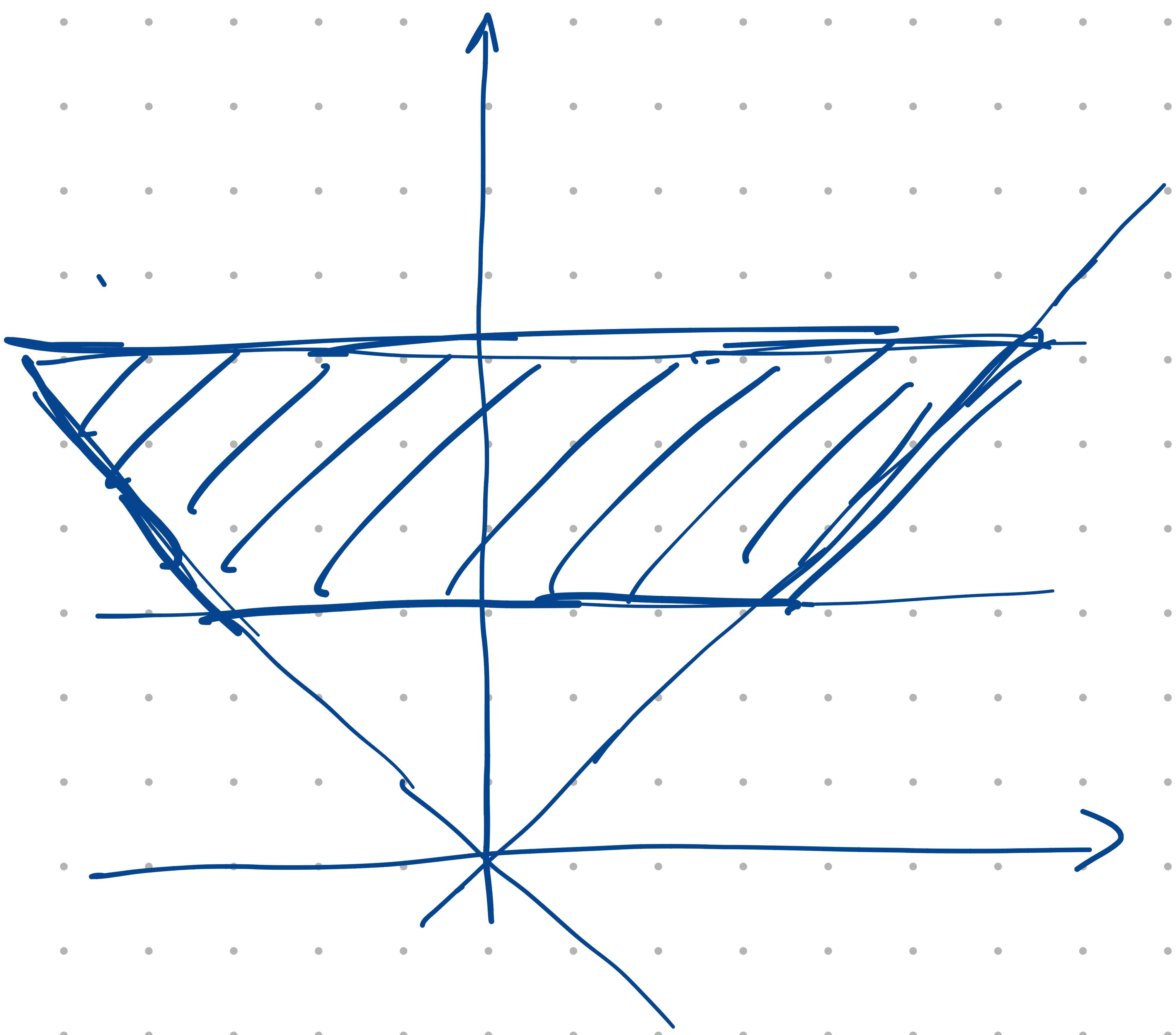


$x \geq 0$

$y \geq 0$

$1 \leq x+y \leq 2$

substitute



$\frac{v-u}{2} \geq 0$

$\frac{v+u}{2} \geq 0$

$1 \leq v \leq 2$